CLASS 7

FORMULATION AND SOLUTION OF ECONOMIC ORDER QUANTITY (EOQ) MODEL

List of symbols used:

We shall use the following symbols for the development of EOQ inventory:

Symbol	Meaning
D	Annual demand (units/year)
Q	Order quantity (units)
C_o	Ordering cost per order
C_h	Holding cost per unit per year

(1) Number of orders per year $= \frac{D}{Q}$

(2) Average inventory $= \frac{Q}{2}$

(3) Ordering Cost (OC)
= Total annual cost of storing inventory
= Number of orders per year × Cost per order
$= \frac{D}{Q} \times C_o$

(4) Holding Cost (HC)
= Total annual cost of placing order
= Average inventory × Holding cost per unit per year
$= \frac{Q}{2} \times C_h$

5. Total Inventory Cost (TC)

$= $ Ordering Cost (OC) + Holding Cost (HC)

$= \left(\frac{D}{Q} \times C_o\right) + \left(\frac{Q}{2} \times C_h\right)$

Our objective is to find Q such that the Total Inventory Cost (TC) is minimum.

We have, Total Inventory Cost= TC $= \left(\frac{D}{Q} \times C_o\right) + \left(\frac{Q}{2} \times C_h\right)$............ (*)

Now by differentiate the above equation with respect to Q, we get:

$$\frac{d(TC)}{dQ} = -\frac{D \times C_o}{Q^2} + \frac{C_h}{2}$$

Now to get the minimum cost, we equate the above derivative to zero as follows:

$$\frac{d(TC)}{dQ} = -\frac{D \times C_o}{Q^2} + \frac{C_h}{2} = 0$$

$$\Rightarrow -\frac{D \times C_o}{Q^2} + \frac{C_h}{2} = 0$$

$$\Rightarrow \frac{C_h}{2} = \frac{D \times C_o}{Q^2}$$

$$\Rightarrow Q^2 = \frac{2 \times D \times C_o}{C_h}$$

$\Rightarrow Q = \sqrt{\frac{2 \times D \times C_o}{C_h}}$ (as Q can not be negative)

This Q $\left(= \sqrt{\frac{2 \times D \times C_o}{C_h}}\right)$ is the Economic Order Quantity (EOQ).

Thus EOQ $= \sqrt{\frac{\mathbf{2 \times D \times C_o}}{\mathbf{C_h}}}$

Let TC_{mim} denote the minimum total inventory cost. Then TC_{mim} can be obtained by substituting Q by **EOQ** in equation (*) as follows:

$$TC = \left(\frac{D}{Q} \times C_o\right) + \left(\frac{Q}{2} \times C_h\right)$$

So, $TC_{mim} = \left(\frac{D}{\mathbf{EOQ}} \times C_o\right) + \left(\frac{\mathbf{EOQ}}{2} \times C_h\right)$

$$\Rightarrow TC_{mim} = \left(\frac{D}{\sqrt{\frac{2\times D\times C_o}{C_h}}}\times C_o\right) + \left(\frac{\sqrt{\frac{2\times D\times C_o}{C_h}}}{2}\times C_h\right)$$

$$\Rightarrow TC_{mim} = \sqrt{\frac{D\times C_o\times C_h}{2}} + \sqrt{\frac{D\times C_o\times C_h}{2}}$$

$$\Rightarrow TC_{mim} = 2\times\sqrt{\frac{D\times C_o\times C_h}{2}}$$

$$\Rightarrow TC_{mim} = \sqrt{2\times D\times C_o\times C_h}$$

Thus, the minimum total inventory cost = $TC_{mim} = \sqrt{2\times D\times C_o\times C_h}$

Let N denote the number of orders per year under EOQ model.

Then, $N = \frac{\text{Total annual demand}}{\text{Optimal order size}} = \frac{D}{EOQ} = \frac{D}{\sqrt{\frac{2\times D\times C_o}{C_h}}}$

Thus, N = Number of orders per year under EOQ model= $\frac{D}{EOQ} = \sqrt{\frac{D\times C_h}{2\times C_o}}$

Let T denote the time between orders under EOQ model.

Then, $T = \frac{EOQ}{D} = \frac{\sqrt{\frac{2\times D\times C_o}{C_h}}}{D} = \sqrt{\frac{2\times C_o}{D\times C_h}}$

Thus, T = Time between orders under EOQ model= $\frac{EOQ}{D} = \sqrt{\frac{2\times C_o}{D\times C_h}}$

Thus, under EOQ model, we have:

Symbol	Meaning	Formula
D	Annual demand (units/year)	-
C_o	Ordering cost per order	-
C_h	Holding cost per unit per year	-
EOQ	Economic Order Quantity	$\mathbf{EOQ} = \sqrt{\frac{2 \times D \times C_o}{C_h}}$ -
N	Number of orders per year under EOQ model	$N = \frac{D}{EOQ} = \sqrt{\frac{D \times C_h}{2 \times C_o}}$
T	Time between orders under EOQ model	$T = \frac{EOQ}{D} = \sqrt{\frac{2 \times C_o}{D \times C_h}}$
OC	Ordering Cost under EOQ model	$OC = \frac{D}{EOQ} \times C_o$ -
HC	Holding (Carrying) Cost under EOQ model	$HC = \frac{EOQ}{2} \times C_h$ -
TC or TC_{mim}	Total Inventory Cost under EOQ model Or Minimum Total Inventory Cost	$TC = TC_{mim} = (OC + HC)$ $= \left(\frac{D}{EOQ} \times C_o + \frac{EOQ}{2} \times C_h\right)$ $= \sqrt{2 \times D \times C_o \times C_h}$