(1)

Crammer's Rule

Solutions of three linear equations having three unknowns with the help of determinents

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

where x, y, z are unknows. But $a_1, b_1, c_1, \ldots, c_3$ and d_1, d_2, d_3 are known quantities.

The solutions of the equations with the help of determinents are

$$x = \frac{1}{\Delta}\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}; \quad y = \frac{1}{\Delta}\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$$

and $$Z = \frac{1}{\Delta}\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$ where $$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0$$

Working rule:

(i) Find Δ $(\neq 0)$ (For $\Delta = 0$, crammer's rule is not applicable)

(ii) In finding the value of x, replacing the coefficient of x in the first column by the column or constants of right side, find the determinent and divide the result thus obtained by the value of Δ.

(iii) Similar treatments for the value of y and z

(ii) ②

$2x + 3y = 13$

$x + 7y = 23$

Ans: Here,

$$\Delta = \begin{vmatrix} 2 & 3 \\ 1 & 7 \end{vmatrix}$$

$= 14 - 3$

$= 11 \neq 0$

Solving by Crammer's rule we have -

$$\frac{x}{\begin{vmatrix} 13 & 3 \\ 23 & 7 \end{vmatrix}} = \frac{y}{\begin{vmatrix} 2 & 13 \\ 1 & 23 \end{vmatrix}} = \frac{1}{\Delta}$$

$$\Rightarrow \frac{x}{91 - 69} = \frac{y}{46 - 13} = \frac{1}{11}$$

$$\Rightarrow \frac{x}{22} = \frac{y}{33} = \frac{1}{11}$$

$\therefore \frac{x}{22} = \frac{1}{11}$, $\frac{y}{33} = \frac{1}{11}$

$\Rightarrow 11x = 22$ $\Rightarrow 11y = 33$

$\Rightarrow x = \cancel{\frac{22}{11}}\,2$ $\Rightarrow y = \cancel{\frac{33}{11}}\,3$

$= 2$ $= 3$

$\left.\begin{array}{l} x = 2 \\ y = 3 \end{array}\right\}$ Ans

(iii) $x - 2y - 8 = 0$

$7x + 3y - 5 = 0$

Ans: The equation can be written as —

$$x - 2y = 8 \longrightarrow (\text{i})$$
$$7x + 3y = 5 \longrightarrow (\text{ii})$$

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Here,
$$\Delta = \begin{vmatrix} 1 & -2 \\ 7 & 3 \end{vmatrix}$$
$$= 3 - (-14)$$
$$= 3 + 14$$
$$= 17$$

Solving by crammers rule we have —

$$\frac{x}{\begin{vmatrix} 8 & -2 \\ 5 & 3 \end{vmatrix}} = \frac{y}{\begin{vmatrix} 1 & 8 \\ 7 & 5 \end{vmatrix}} = \frac{1}{\Delta}$$

$$\Rightarrow \frac{x}{24 - (-10)} = \frac{y}{5 - 56} = \frac{1}{17}$$

$$\Rightarrow \frac{x}{34} = \frac{y}{-51} = \frac{1}{17}$$

$$\therefore \frac{x}{34} = \frac{1}{17}, \quad \frac{y}{-51} = \frac{1}{17}$$

$$\therefore x = 2, \quad y = -3$$

$\left.\begin{matrix} x = 2 \\ y = -3 \end{matrix}\right\}$ **Ans:**

6. Solve by Crammer's rule — (4)

(1) $x + 2y + 3z - 6 = 0$

$2x + 4y - 7 = -z$

$3x + 2y + 9z - 14 = 0$

Ans: The equation can be written as —

$$x + 2y + 3z = 6 \longrightarrow (i)$$

$$2x + 4y + z = 7 \longrightarrow (ii)$$

$$3x + 2y + 9z = 14 \longrightarrow (iii)$$

Here,

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 1 \\ 3 & 2 & 9 \end{vmatrix}$$

$$= 1(36-2) - 2(18-3) + 3(4-12)$$

$$= 34 - 30 - 24$$

$$= 34 - 54$$

$$= -20 \neq 0$$

Solving by crammer's rule we have —

$$\frac{x}{\begin{vmatrix} 6 & 2 & 3 \\ 7 & 4 & 1 \\ 14 & 2 & 9 \end{vmatrix}} = \frac{y}{\begin{vmatrix} 1 & 6 & 3 \\ 2 & 7 & 1 \\ 3 & 14 & 9 \end{vmatrix}} = \frac{z}{\begin{vmatrix} 1 & 2 & 6 \\ 2 & 4 & 7 \\ 3 & 2 & 14 \end{vmatrix}} = \frac{1}{\Delta}$$

$$\Rightarrow \frac{x}{6(36-2)-2(63-14)+3(14-56)} = \frac{y}{1(63-14)-6(18-3)+3(28-21)} = \frac{z}{1(56-14)-2(28-21)+6(4-12)} = \frac{1}{-20}$$

$$\Rightarrow \frac{x}{6.34-2.49+3(-42)} = \frac{y}{1.49-6.15+3.7} = \frac{z}{1.42-2.7+6(-8)} = \frac{1}{-20}$$

$$\Rightarrow \frac{x}{204-98-126} = \frac{y}{49-90+21} = \frac{z}{42-14-48} = \frac{1}{-20}$$

$$\Rightarrow \frac{x}{-20} = \frac{y}{-20} = \frac{z}{-20} = \frac{1}{-20}$$

$$\therefore \frac{x}{-20} = \frac{1}{-20}, \quad \frac{y}{-20} = \frac{1}{-20}, \quad \frac{z}{-20} = \frac{1}{-20}$$

$$\Rightarrow -20x = -20 \qquad \Rightarrow -20y = -20 \qquad \Rightarrow -20z = -20$$

$$\Rightarrow x = \frac{20}{20} = 1 \qquad \Rightarrow y = \frac{20}{20} = 1. \qquad \Rightarrow z = \frac{20}{20} = 1$$

$$\left.\begin{aligned} x &= 1 \\ y &= 1 \\ z &= 1 \end{aligned}\right\} \text{Ans.}$$

Solve by Crammer's Rule —

$$3x + y + z = 10$$
$$x + y - z = 0$$
$$5x - 9y = 1$$

Ans: The equation can be written as —

$$3x + y + z = 10 \longrightarrow (i)$$
$$x + y - z = 0 \longrightarrow (ii)$$
$$5x - 9y + 0.z = 1 \longrightarrow (iii)$$

Here,

$$\Delta = \begin{vmatrix} 3 & 1 & 1 \\ 1 & 1 & -1 \\ 5 & -9 & 0 \end{vmatrix}$$

$$= 3(0-9) - 1(0+5) + 1(-9-5)$$

$$= -27 - 5 - 14$$

$$= -46 \neq 0$$

Solving by Crammer's rule we have, (7)

$$\frac{x}{\begin{vmatrix}10 & 1 & 1\\ 0 & 1 & -1\\ 1 & -9 & 0\end{vmatrix}} = \frac{y}{\begin{vmatrix}3 & 10 & 1\\ 1 & 0 & -1\\ 5 & 1 & 0\end{vmatrix}} = \frac{z}{\begin{vmatrix}3 & 1 & 10\\ 1 & 1 & 0\\ 5 & -9 & 1\end{vmatrix}} = \frac{1}{\Delta}$$

$$\Rightarrow \frac{x}{10(0-9)-1(0+1)+1(0-1)} = \frac{y}{3(0+1)-10(0+5)+1(1-0)} = \frac{z}{3(1+0)-1(1-0)+10(-9-5)} = \frac{1}{-46}$$

$$\Rightarrow \frac{x}{-90-1-1} = \frac{y}{3-50+1} = \frac{z}{3-1-140} = \frac{1}{-46}$$

$$\Rightarrow \frac{x}{-92} = \frac{y}{-46} = \frac{z}{-138} = \frac{1}{-46}$$

$$\therefore \frac{x}{-92} = \frac{1}{-46}, \quad \frac{y}{-46} = \frac{1}{-46}, \quad \frac{z}{-138} = \frac{1}{-46}$$

$$\Rightarrow -46x = -92, \quad \Rightarrow -46y = -46, \quad \Rightarrow -46z = -138$$

$$\Rightarrow x = \frac{92}{46} \quad \Rightarrow y = \frac{46}{46} \quad \Rightarrow z = \frac{138}{46}$$

$$= 2 \qquad = 1 \qquad = 3$$

$$\therefore \left.\begin{aligned} x &= 2\\ y &= 1\\ z &= 3 \end{aligned}\right\} \text{Ans}$$