$\boxed{\hat{y} = \alpha + \beta x_i + u_i}$

Assumptions for random term (u) for applying OLS method :-

1. ~~Et~~ U is purely a random term value of u depends upon in chances.

2. The mean value of u is always '0'

i.e mean of $u = \Sigma(u_i) = 0$

U can take + & − value so that their sum is 0.

3. The variance of u for different values of x always remain constant i.e -

variance of $u_i = E\{u_i - E(u_i)\}^2 =$ Constant let σ_u^2

4. The values of u for any particular value of x is normally distributed i.e, $u_i \sim N(0, \sigma_u^2)$.

5. The different values of u are not correlated with each other i.e $E(u_i, u_j) = 0$

$\text{Covariance}(u_i u_j) = 0$

6. the random term u is not correlated with the explanatory variable X i.e $\text{cov}(u_i x_i) = 0$ $E(u_i x_i) = 0$

other assumptions

ⓐ The values of x remain the same in repeated samples.

VIMP ⓑ Different explanatory variables are not correlated with each other i.e. $E(x_i x_j) = 0$

~~other assumptions~~

ⓒ The data is measured correctly.

ⓓ Model is correctly specified.

ⓔ The model is identified.

OLS Method

The method of OLS is attributed to Carl Friedrich Gauss, a German Mathematician.

The two variable PRF

$$Y_i = \alpha + \beta X_i + u_i$$

The PRF is not directly observable. we estimate it from SRF

$$Y_i = \hat{\alpha} + \hat{\beta} X_i + \hat{u}_i$$

$$= \hat{y}_i + \hat{u}_i$$ [$\because \hat{y}$ is the estimated value of Y_i]

Now $\hat{u}_i = y_i - \hat{y}_i$

$$= Y_i - \hat{\alpha} - \hat{\beta} X_i$$

which shows that $\hat{u}_i$ (the residuals) are simply the difference between actual and estimated Y values.

Now given n pairs of observations on Y and X, we would like to determine SRF in such a manner that it is as close as possible to the actual Y. i.e the sum of residuals $\Sigma \hat{u}_i = \Sigma(Y_i - \hat{Y}_i)$ is as small as possible.

It is not good criterion if we adopt the criterion of minimising $\Sigma \hat{u}_i$ because the algebric sum of these residuals is zero (i.e, $\Sigma \hat{u}_i = 0$). We can avoid this ~~tea~~ problem if we adopt the least-squares criterion.

$$\Sigma \hat{u}_i^2 = \Sigma (Y_i - \hat{Y}_i)^2$$

$$= \Sigma (Y_i - \hat{\alpha} - \hat{\beta} X_i)^2$$

Now $\Sigma \hat{u}_i^2 = f(\hat{\alpha}, \hat{\beta})$

The method of least squares provides us with unique estimates of $\hat{\alpha}$ and $\hat{\beta}$ that give the smallest possible value of $\Sigma \hat{u}_i^2$.

Differentiating $\Sigma \hat{u}_i^2$ by α and β we get —

$$\frac{d \Sigma u_i^2}{d\hat{\alpha}} = \frac{d}{d\alpha} \Sigma (Y_i - \hat{\alpha} - \hat{\beta} X_i)^2 = 0$$

$$\Rightarrow -2 \Sigma (Y_i - \hat{\alpha} - \hat{\beta} X_i) = 0$$

$$\Rightarrow \Sigma Y_i - n\hat{\alpha} - \hat{\beta} \Sigma X_i = 0 \quad \text{——— (i)}$$

(ii) $\frac{d\Sigma u_i^2}{d\hat{\beta}} = \frac{d}{d\hat{\beta}} \Sigma(y_i - \hat{\alpha} - \hat{\beta} x_i)^2 = 0$

$\Rightarrow -2\Sigma(y_i - \hat{\alpha} - \hat{\beta} x_i) . -x_i = 0$

$\Rightarrow 2x_i \Sigma(y_i - \hat{\alpha} - \hat{\beta} x_i) = 0$

$\Rightarrow \Sigma x_i y_i - \hat{\alpha}\Sigma x_i - \hat{\beta}\Sigma x_i^2 = 0$ —— (ii)

Now, dividing (i) by n, we get ——

$\Rightarrow \frac{\Sigma y_i}{n} - \frac{\cancel{n}\hat{\alpha}}{\cancel{n}} - \frac{\hat{\beta}\Sigma x_i}{n} = 0$

$\Rightarrow \bar{Y} - \hat{\alpha} - \hat{\beta}\bar{X} = 0$

$\boxed{\Rightarrow \hat{\alpha} = \bar{Y} - \hat{\beta}\bar{X}}$

Now, putting $\hat{\alpha} = \bar{Y} - \hat{\beta}\bar{X}$, in (ii) we get ——

$\Rightarrow \Sigma x_i y_i - (\bar{Y} - \hat{\beta}\bar{X})\Sigma x_i - \hat{\beta}\Sigma x_i^2 = 0$

$\Rightarrow \Sigma x_i y_i - n\bar{Y}\bar{X} + n\hat{\beta}\bar{X}^2 - \hat{\beta}\Sigma x_i^2 = 0$

$\Rightarrow \Sigma x_i y_i - n\bar{Y}\bar{X} - \hat{\beta}(\Sigma x_i^2 - n\bar{X}^2) = 0$

$\Rightarrow \hat{\beta} = \frac{\Sigma x_i y_i - n\bar{Y}\bar{X}}{\Sigma x_i^2 - n\bar{X}^2}$

$$\Rightarrow \hat{\beta} = \frac{\Sigma x_i y_i - n \cdot \frac{\Sigma y_i}{n} \frac{\Sigma x_i}{n}}{\Sigma x_i^2 - n \cdot \left(\frac{\Sigma x}{n}\right)^2}$$

$$\Rightarrow \hat{\beta} = \frac{\frac{n \Sigma x_i y_i - \Sigma x_i \Sigma y_i}{n}}{\frac{n \Sigma x_i^2 - (\Sigma x_i)^2}{n}}$$

$$\Rightarrow \hat{\beta} = \frac{n \Sigma x_i y_i - \Sigma x_i \Sigma y_i}{n \Sigma x_i^2 - (\Sigma x_i)^2}$$

$$\Rightarrow \hat{\beta} = \frac{\Sigma (x_i - \bar{x})(y_i - \bar{y})}{\Sigma (x - \bar{x})^2}$$

$$\boxed{\Rightarrow \hat{\beta} = \frac{\Sigma x_i y_i}{\Sigma x_i^2}}$$